

WEIGHTED ESTIMATES FOR HARDY-LITTLEWOOD MAXIMAL FUNCTIONS ON HARMONIC NA GROUPS

TAPENDU RANA

Abstract:

For a locally integrable function f on \mathbb{R}^d , its Hardy-Littlewood maximal function $M_{\mathbb{R}^d} f$ is defined as

$$M_{\mathbb{R}^d} f = \sup_{r>0} \frac{1}{|B(r, x)|} \int_{B(r, x)} |f(y)| dy$$

It is well-known that the operator is bounded on L^p for $p > 1$ and of weak type $(1, 1)$. In the seminal works of Fefferman-Stein and Muckenhoupt, they provided a characterization of weights on \mathbb{R}^d by introducing the classical A_p weight condition, which allows the weighted boundedness of the Hardy-Littlewood maximal operator.

In this talk, we will discuss the weighted boundedness of the Hardy-Littlewood maximal operator in Harmonic N A groups, also known as Damek-Ricci spaces. More precisely, we will demonstrate that the Euclidean type A_p condition is not necessary for the Hardy-Littlewood maximal operator to be bounded in this setting, making it apparent that such conditions are not suitable in Harmonic N A groups. We provide a necessary condition and define a suitable notion of admissible A_p class of weights for which the maximal operator is weighted bounded. Furthermore, as an endpoint case, we will prove a variant of the Fefferman-Stein inequality.

This talk is based on a joint work with Pritam Ganguly and Jayanta Sarkar.